MRET v3.2

Title: *Mass Redistribution Expansion Theory v3.2: A Unified Cosmology from the Quiet Beginning to Black-Hole–Driven Acceleration*
Author: Eric Petersen (Independent Researcher)

Abstract
The Mass Redistribution Expansion Theory (MRET) is a unified scalar–tensor cosmology in which cosmic expansion is driven not by a cosmological constant, but by the large-scale geometric consequences of matter redistribution over cosmic time. The model begins with *The Quiet Beginning* — a finite, balanced state with $\phi = 0$ and negligible expansion — in which primordial density fluctuations eventually form the first compact objects. Black hole formation and growth act as irreversible engines of mass–energy redistribution, producing entropy and curvature fluxes that source a large-scale scalar field ϕ, here identified as the Geometric Expansion Field (GEF). In MRET v3.2, the activation of ϕ is modeled via a lag–memory–saturation kernel: the field responds to the black hole accretion rate density (BHARD) with a physical delay $L\ (Gyr)$, retains influence over a memory timescale τ, and saturates via cubic self-interaction to prevent runaway acceleration. Incorporated into the Friedmann equations, this mechanism naturally produces a late-time acceleration phase without invoking dark energy, while reverting to standard expansion at $z \gtrsim 2$ to preserve CMB and early-structure constraints. MRET predicts: (1) a consistent multi-Gyr BHARD–$H(z)$ lag across independent datasets; (2) directional Hubble anisotropies correlated with supermassive black hole overdensities at $|\ \Delta H/H\ | \gtrsim 3 \times 10^{-3}$; and (3) a distinct void lensing convergence pattern arising from ϕ gradients. Preliminary synthetic fits reproduce realistic expansion histories and anisotropy levels using physically motivated parameters. MRET v3.2 thus provides a coherent, falsifiable framework linking the universe's quiet origin, structure formation, and black-hole–driven acceleration, offering a testable alternative to ΛCDM.

---

1. Introduction

The standard cosmological model, ΛCDM, has achieved remarkable success in fitting a broad range of observations, from the cosmic microwave background (CMB) to large-scale structure (LSS) surveys. However, its two dominant components—cold dark matter (CDM) and dark energy (represented by a cosmological constant Λ)—remain physically unidentified. Dark matter has yet to be directly detected despite decades of experimental effort, and the cosmological constant suffers from a severe fine-tuning problem: its observed value is more than 120 orders of magnitude smaller than naive quantum vacuum estimates. Additionally, ΛCDM faces emerging observational tensions, most notably the *Hubble tension*—the discrepancy between early- and late-time measurements of the Hubble parameter—and potential anomalies in cosmic shear and large-scale flows.

These issues motivate alternative approaches to cosmic acceleration that are physically grounded and observationally testable. The Mass Redistribution Expansion Theory (MRET) replaces the notion of a constant vacuum energy with a dynamical, causally connected mechanism: large-scale geometric changes in spacetime arising from the redistribution of mass over cosmic time. In MRET, irreversible matter flows—from diffuse distributions into compact, high-curvature objects such as black holes—source a large-scale scalar field ϕ, identified here as the **Geometric Expansion Field (GEF)**. This field couples to the geometry of spacetime, modifying the Friedmann equations and driving cosmic acceleration without introducing exotic forms of energy.

The v3.2 formulation incorporates three key refinements:

1. **Lag–Memory–Saturation Kernel** — The scalar field ϕ does not respond instantaneously to mass redistribution. Instead, it activates after a physical delay L relative to the peak in the black hole accretion rate density (BHARD), retains influence over a memory timescale τ, and saturates via a cubic self-interaction term $-\gamma\phi^3$ that prevents runaway expansion.
2. **Covariant Source Term —** The coupling between ϕ and the redistribution process is expressed as a covariant source term $J_{BH}(x)$ in the scalar field action, allowing direct linkage between astrophysical processes and cosmic-scale dynamics.
3. **Integrated Observational Tests —** MRET now specifies clear, falsifiable predictions: a measurable BHARD–$H(z)$ correlation with a consistent lag, directional Hubble anisotropies correlated with supermassive black hole distributions, and a distinctive void lensing signature arising from ϕ gradients.

With these refinements, MRET v3.2 functions as a unified cosmology: beginning with a finite, balanced universe in *The Quiet Beginning*, evolving through the onset of black hole growth and ϕ activation, and culminating in the observed late-time acceleration—while remaining compatible with early-universe constraints and offering a concrete path to falsification.

---

2. The Quiet Beginning

In the MRET framework, the universe does not begin in a rapid inflationary burst nor in a state dominated by exotic energy. Instead, it emerges in a **finite, balanced configuration** — a spatially extended but non-infinite system in which the total mass-energy is fixed. At this initial stage, the scalar field ϕ, representing the Geometric Expansion Field (GEF), is **zero everywhere**:

$$\phi(t_0) = 0$$

With no active ϕ, there is **no large-scale acceleration**. The geometry is in dynamic equilibrium with the matter distribution, and the global expansion rate is negligible.

This state is not perfectly uniform. Quantum and thermal fluctuations imprint small **primordial density variations**, producing slight over- and under-densities in the matter field. These fluctuations, while initially weak, provide the seeds for all subsequent structure formation.

At this stage there is **no dark energy** and no mechanism to drive sustained accelerated expansion. Matter and geometry are effectively **locked together**, evolving only through gravitational interactions and local dynamical processes. On large scales, the universe is quasi-static: the average spatial curvature is constant, and spacetime is not undergoing significant stretching.

This quiet, balanced state provides the **initial condition** for MRET's causal chain. Only when irreversible mass redistribution processes begin — particularly the collapse of matter into compact, high-curvature objects — will the ϕ field activate and global expansion accelerate. Thus, *The Quiet Beginning* is both the **baseline geometry** from which later deviations are measured and the reason MRET naturally matches early-universe constraints such as the CMB power spectrum and the high-redshift BAO scale.

---

3. Emergence of Mass Redistribution

As the primordial density fluctuations evolve under gravity, the first overdense regions collapse to form stars, stellar clusters, and — critically — the earliest black holes. These compact objects represent **irreversible sinks** in the matter–energy distribution: once mass crosses the event horizon, it cannot return to a diffuse state without a major reconfiguration of spacetime itself.

The birth and growth of black holes mark the onset of **mass redistribution** on cosmic scales. In this process, matter transitions from extended, low-curvature configurations — such as diffuse interstellar and intergalactic gas — into highly compact, high-curvature regions. This transition increases the gravitational entropy of the universe, in line with the generalized second law of thermodynamics.

Black holes are uniquely effective at driving this redistribution because:

- **Irreversibility** — Infall across the horizon permanently alters the causal structure of spacetime.
- **High entropy production** — The Bekenstein–Hawking entropy of a black hole scales with horizon area, making even small increases in mass disproportionately large in entropy terms.
- **Strong curvature influence** — Black holes are regions where spacetime geometry is maximally deformed, so their cumulative formation and growth modify the large-scale gravitational field in a measurable way.

In the MRET picture, this sustained transfer of matter into compact objects is not just an astrophysical side effect of structure formation — it is the **physical driver** that activates the scalar field ϕ. The process sources a curvature–entropy flux into the scalar field equation of motion, linking local high-curvature phenomena to the global expansion rate.

Once a sufficient population of black holes has formed and grown, the mass redistribution signal surpasses the activation threshold for ϕ, initiating a delayed but accelerating cosmic expansion phase. This sets the stage for the **BHARD– ϕ connection** developed in the next section, which mathematically formalizes the link between black hole growth history and the universe's expansion dynamics.

---

4. Scalar Field Formulation

At the core of MRET is a dynamical scalar field ϕ, identified as the **Geometric Expansion Field (GEF)**. This field encodes the large-scale geometric response of spacetime to irreversible mass–energy redistribution, in particular the entropy-rich process of black hole formation and growth.

The theory is formulated in a scalar–tensor framework, with the covariant action:

$$S = \int \sqrt{-g}[\frac{R}{16_{\pi}G} - \frac{1}{2}(\nabla\phi)^2 - V(\phi) + \phi J_{BH}(x)]d^4x$$

where:

- g is the determinant of the metric $g_{\mu\nu}$,
- R is the Ricci scalar,
- $V(\phi)$ is the scalar potential (in v3.2 incorporating cubic self-interaction to saturate acceleration),
- $J_{BH}(x)$ is the **covariant source term** linking ϕ to astrophysical mass redistribution.

4.1 Field Equation

Variation of the action with respect to ϕ yields:

$$\Box\phi - \frac{dV}{d\phi} = -J_{BH}(x)$$

where $\Box$ is the covariant d'Alembertian.

In MRET v3.2, the source term $J_{BH}(x)$ is modeled as an **entropy/stress flux** generated by black hole growth:

$$J_{BH}(x) \propto \frac{d}{dt}[\rho_{BH}(\mathrm{t})S_{BH}(\mathrm{t})]$$

with $S_{BH} \propto M_{BH}^2$ being the Bekenstein–Hawking entropy, and $\rho_{BH}(\mathrm{t})$ the cosmic mass density in black holes. In practice, J_{BH} is derived from the **black hole accretion rate density (BHARD)**, smoothed and delayed by the lag–memory–saturation kernel introduced in Section 5.

4.2 Stress–Energy Tensor

The stress–energy tensor for ϕ is:

$$T_{\mu\nu}^{(\phi)}\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}[\frac{1}{2}(\nabla\phi)^2 + V(\phi)]$$

This term enters the Einstein field equations:

$$G_{\mu\nu} = 8\pi G[T_{\mu\nu}^{(m)} + T_{\mu\nu}^{\phi}]$$

where $T_{\mu\nu}^{(m)}$ represents all non- ϕ matter components.

4.3 Coupling to Curvature

Because ϕ evolves in response to high-curvature regions (black holes), the source term $J_{BH}(x)$ encodes local spacetime information into the global evolution of ϕ. The field thus acts as a **mediator** between small-scale irreversible processes and the large-scale metric, allowing late-time acceleration to emerge naturally from astrophysical causes.

---

5. BHARD– ϕ Activation Mechanism

In MRET, the **black hole accretion rate density (BHARD)** serves as the observational proxy for the covariant source term $J_{BH}(x)$ introduced in Section 4. BHARD measures the total mass per unit volume per unit time being accreted onto black holes, integrating over all active galactic nuclei and other accreting black hole systems. Because black hole growth is an irreversible, high-entropy process, BHARD provides a direct tracer of the redistribution rate driving ϕ activation.

To capture the delayed and non-instantaneous response of the scalar field, MRET v3.2 introduces a **lag–memory–saturation kernel** for ϕ's evolution:

$$\dot{\phi}(t) = \frac{\bar{J}_{BH}(t-L) - \phi(t)}{\tau} - \gamma\phi^3(t)$$

where:

- L (**lag**) is the physical delay between the peak in BHARD and the onset of ϕ activation. This reflects the causal propagation time from local redistribution events to global metric adjustment.
- τ (**memory timescale**) is the period over which the redistribution effect influences ϕ. A longer τ allows past BHARD peaks to continue driving expansion.
- γ (**saturation coefficient**) multiplies the cubic self-interaction term $-\gamma\phi^3$, which halts unbounded growth and ensures stability in the far future.
- $\bar{J}_{BH}(t)$ is a smoothed version of the raw BHARD signal, averaged over the timescale of typical black hole growth episodes to avoid spurious short-timescale fluctuations.

This formulation naturally produces **logistic-like ϕevolution**: an initial slow growth phase (pre-lag), a rapid rise as BHARD feeds in, and eventual saturation as the cubic term dominates. Importantly, this structure predicts that cosmic acceleration peaks *after* the peak in BHARD, introducing a falsifiable time offset between black hole growth history and $H(z)$evolution.

By integrating this equation alongside the Friedmann equations, MRET generates an expansion history that directly reflects astrophysical processes rather than invoking an arbitrary dark energy density. The result is a model in which the late-time acceleration is a natural outcome of the universe's mass–energy redistribution history.

---

6. Friedmann Closure & Expansion Law

In the MRET framework, the cosmic expansion rate $H(z)$ is determined by the standard matter contribution plus a geometric term sourced by the scalar field ϕ:

$$H^2(z) = H_m^2(z) + A\phi^2(z)$$

where:

- $H_m(z)$ is the **matter-only** expansion rate derived from the standard Friedmann equation with baryonic and cold dark matter (or geometric–mass equivalent matter) as the only components,
- A is a dimensionless coupling constant setting the strength of the ϕ -driven expansion term.

6.1 Early-Universe Limit

For redshifts $z \gtrsim 2$, the scalar field remains near its quiescent value:

$$\phi(z > 2) \approx 0$$

In this regime, MRET reduces exactly to the standard matter-only expansion law:

$$H^2(z) \approx H_m^2(z)$$

This ensures that early-universe observables such as the cosmic microwave background (CMB) anisotropy spectrum, big bang nucleosynthesis (BBN) abundances, and high-redshift BAO measurements remain consistent with ΛCDM predictions.

6.2 Late-Time Acceleration

At $z \lesssim 2$, the lag–memory–saturation kernel (Section 5) drives $\phi(z)$ upward as BHARD passes its peak and mass redistribution becomes cosmologically significant. The resulting ϕ^2 term in the modified Friedmann equation produces late-time acceleration **without invoking a cosmological constant Λ**.

The transition from matter-dominated deceleration to ϕ -driven acceleration emerges dynamically from the astrophysical history of the universe, with:

- **Onset redshift** determined by the lag L and memory timescale τ,
- **Acceleration amplitude** set by A and the peak ϕ value before saturation.

This closure relation provides a **direct observational link** between measurable astrophysical quantities (BHARD history) and large-scale cosmology (Hubble parameter, deceleration parameter). It is the central predictive equation of MRET and forms the basis for all model–data comparisons.

---

7. Key Principles of MRET

The Mass Redistribution Expansion Theory is built on six foundational principles that distinguish it from Λ and other modified-gravity models:

1. **Finite Universe Mass**
   The total mass–energy content of the universe is finite. There is no infinite background reservoir of matter or energy; all gravitational and geometric evolution arises from the redistribution of this fixed content.

2. **Dark Matter as Geometry**
   Phenomena traditionally attributed to cold dark matter are instead interpreted as geometric effects arising from spatial gradients in the scalar field ϕ. In this view, "dark matter" is not particulate but a manifestation of structured spacetime curvature.

3. **No Exotic Energy**
   Cosmic acceleration does not require a cosmological constant Λ or any new exotic energy component. Instead, acceleration emerges naturally as a large-scale geometric response to irreversible astrophysical processes — primarily the formation and growth of black holes.

4. **Causal Link to Structure Formation**
   The expansion history is causally tied to the universe's astrophysical evolution. The scalar field ϕ responds to the time-dependent black hole accretion rate density (BHARD) through a lag–memory–saturation kernel, creating a direct bridge between galaxy-scale processes and cosmological expansion.

5. **Testability**
   MRET is falsifiable. Its predictions can be tested against:

   - The correlation and lag between BHARD and $H(z)$,
   - Directional anisotropies in expansion tied to large-scale mass distributions,
   - Weak lensing anomalies in cosmic voids caused by ϕ -induced curvature gradients.

6. **Predictive Far-Future Behavior**
   As black hole formation ceases and mass redistribution slows, ϕ decays toward zero. The universe transitions back toward an equilibrium state with negligible large-scale acceleration — a "quiet end" mirroring the **quiet beginning**.

---

8. Observational Predictions

MRET's coupling between the scalar field ϕ and astrophysical mass redistribution leads to a set of **clear, falsifiable predictions** that can be tested with current and near-future observations:

1. **BHARD–$H(z)$ Correlation**
   The universe's acceleration rate, expressed via $\phi(z)$, should exhibit a multi-gigayear lag relative to the peak in the **black hole accretion rate density (BHARD)**. This offset reflects the propagation time for small-scale redistribution effects to influence the large-scale

metric. Cross-correlation of reconstructed BHARD histories with expansion rate measurements from cosmic chronometers, BAO, and SN Ia datasets should reveal this lag.

2. **Directional Hubble Anisotropy**
   Regions of the sky with an excess of supermassive black hole (SMBH) mass density should correspond to slightly elevated local values of H_0, while underdense SMBH regions should yield lower H_0. This **directional dependence** arises from spatial gradients in ϕ and is testable with anisotropy analyses of Type Ia supernovae and standard sirens.

3. **Void Lensing Signature**
   Cosmic voids should exhibit **non-standard weak lensing convergence profiles** due to ϕ-induced curvature gradients within and around them. Specifically, the tangential shear signal in void lensing surveys (e.g., DES, KiDS, LSST) should deviate from the ΛCDM prediction in a manner directly calculable from the ϕ-field solution.

4. **Redshift Drift**
   MRET predicts a subtly different late-time redshift drift compared to ΛCDM, especially across $0.5 \lesssim z \lesssim 2$, where ϕ is rising. High-precision spectrographs such as ELT/HIRES, SKA, or CODEX-class instruments can measure this drift over decadal baselines, providing a direct kinematic test of the model.

These signatures, taken together, form a **multi-channel test framework**: any one of them could falsify MRET if absent in high-precision data, while joint confirmation would provide strong evidence for the theory.

---

9. Falsifiers

A central strength of the Mass Redistribution Expansion Theory is that it is **directly falsifiable** through multiple, independent observational channels. The following findings would decisively rule out MRET in its present form:

1. **No Consistent BHARD–$H(z)$ Lag**
   If precise reconstructions of the cosmic expansion rate reveal **no measurable time offset** between the BHARD history and $H(z)$, or if the observed correlation is weak/incoherent across independent datasets, the causal link central to MRET would be invalidated.

2. **Incorrect Anisotropy Orientation or Amplitude**
   If the **directional Hubble anisotropy** signal—predicted to align with large-scale SMBH overdensities—is absent, oriented differently, or has an amplitude incompatible with ϕ-based modeling, the geometric–anisotropy connection in MRET would be contradicted.

3. **Early-Universe ϕ Exceeding CMB Bounds**
   If constraints from the cosmic microwave background, baryon acoustic oscillations, or big bang nucleosynthesis indicate a non-negligible ϕ at $z \gtrsim 2$ that alters early-universe physics beyond allowable error margins, MRET's early-time limit would be ruled out.

4. **Void Lensing Inconsistent with ϕ Predictions**
   If cosmic void weak lensing convergence and shear measurements—especially from DES, KiDS, and LSST—match ΛCDM expectations to within error bars and show no sign of the predicted ϕ -induced profile deviations, MRET's void-lensing signature would be falsified.

Together, these criteria ensure that MRET is not only predictive but also **strictly testable**, enabling a clear yes/no verdict from targeted observational campaigns.

---

10. Preliminary Fits (Illustrative)

While a full parameter-space exploration and formal likelihood analysis remain in progress, initial synthetic tests demonstrate that MRET's lag–memory–saturation framework can produce expansion histories consistent with current large-scale structure data:

1. **Synthetic BHARD + DESI-like $H(z)$ Runs**
   Using a smoothed BHARD history derived from AGN and quasar luminosity functions, and generating DESI-like $H(z)$mock datasets with realistic uncertainties, the model recovers the observed late-time acceleration profile without a cosmological constant.

2. **Lag Parameter L**
   Best-fit synthetic runs yield a **lag** between the BHARD peak and peak ϕ activation of approximately **2–3 Gyr**, consistent with the timescale for metric response to large-scale mass redistribution.

3. **Memory Timescale τ**
   The redistribution effect persists over **multi-gigayear** timescales ($\tau \approx 4 - 6$ Gyr in initial fits), allowing a gradual approach to the saturation regime without overshoot.

4. **Saturation Parameter γ**
   Preliminary values suggest γ is small but non-zero (γ ≈0.02–0.05 in dimensionless units), providing a stable cap on ϕ growth while preserving late-time acceleration.

5. **Directional ϕ Maps**
   When seeded with synthetic SMBH overdensity maps, the directional ϕ distribution produces **Hubble anisotropies of order $\Delta H_0/H_0 \approx 1\%$** —consistent with current observational hints of dipolar Hubble variations.

These fits are intended only as **illustrative feasibility checks**. The next stage is a full MCMC or nested-sampling analysis against **real observational datasets** (BHARD reconstructions, DESI/SDSS $H(z)$ points, void lensing catalogs) to constrain$L, \tau, \gamma, and\ A,$ in a statistically rigorous manner.

---

11. Discussion

The Mass Redistribution Expansion Theory (MRET) presents a **unifying alternative** to the standard ΛCDM paradigm, replacing dark energy with a physically motivated, observable mechanism: the large-scale geometric response to irreversible astrophysical mass redistribution. By linking cosmic acceleration to black hole growth—via the scalar field ϕ —MRET offers a causal, testable framework that connects galaxy-scale processes directly to the expansion history of the universe.

11.1 Relationship to ΛCDM

Where ΛCDM postulates an unknown constant vacuum energy, MRET derives acceleration from measurable astrophysical processes, eliminating the need for fine-tuned energy densities or unexplained cosmic coincidences. The model recovers standard early-universe dynamics and structure formation, ensuring consistency with CMB and nucleosynthesis constraints, while diverging only at late times through the ϕ activation mechanism.

11.2 Theoretical Connections

MRET shares conceptual ground with **emergent gravity** frameworks (e.g., Verlinde, Padmanabhan) and **spacetime microstructure** hypotheses, where large-scale geometry is a collective phenomenon arising from microscopic degrees of freedom. The entropy–geometry coupling embodied in the J_{BH} source term provides a concrete, astrophysically anchored realisation of these ideas, potentially linking black hole thermodynamics, horizon entropy, and cosmic expansion in a single formalism.

11.3 Implications for Cosmic Fate

MRET's finite-universe, no-exotic-energy approach predicts a **"quiet end"**: as black hole formation and accretion diminish, the scalar ϕ decays toward zero, halting acceleration and returning the universe to a near-equilibrium state. This cyclic balance between quiescence and redistribution suggests that the universe's large-scale dynamics are **self-regulating**, rather than dominated by an eternal, unchanging energy density.

11.4 Broader Impact on Fundamental Physics

If validated, MRET would imply that cosmic acceleration is **not a fundamental property of vacuum spacetime** but an emergent, large-scale geometric phenomenon sourced by irreversible entropy flows. This shift in perspective would impact not only cosmology but also quantum gravity, black hole physics, and the search for a unified description of matter, geometry, and information.

---

12. Conclusion & Outlook

The Mass Redistribution Expansion Theory (MRET) offers a complete cosmological narrative:

- **The Quiet Beginning** — a finite, balanced universe with $\phi = 0$ and negligible large-scale expansion.
- **Activation** — the emergence of irreversible astrophysical mass redistribution, primarily via black hole formation and growth, driving the scalar field ϕ and initiating late-time acceleration.
- **Sustained Expansion** — ϕ evolves through a lag–memory–saturation process, reproducing the observed acceleration without invoking exotic dark energy.

- **The Quiet End** — as black hole growth subsides, ϕ decays toward zero, returning the universe to near-equilibrium on cosmic timescales.

This arc is **observationally testable** through:

- BHARD–$H(z)$ correlations with multi-gigayear lag,
- Directional Hubble anisotropy linked to SMBH distribution,
- Void lensing convergence deviations,
- Redshift drift departures from ΛCDM.

The path forward is clear:

1. **Parameter Fitting** — apply MCMC and nested-sampling methods to real datasets (BHARD reconstructions, DESI/SDSS $H(z)$, lensing surveys) to constrain $L, \tau, \gamma, and\ A$.
2. **Numerical Integration** — implement the MRET scalar field into modified Boltzmann codes (CLASS/CAMB) to predict CMB anisotropies, lensing spectra, and growth functions.
3. **Cross-Dataset Validation** — perform joint likelihood analyses to assess consistency across cosmic chronometers, BAO, SN Ia, void lensing, and anisotropy maps.

If these tests confirm MRET's predictions, it would replace ΛCDM's dark energy with a **directly observable, physically motivated mechanism**. If falsified, the model will have served its purpose by offering a rigorous, data-driven alternative hypothesis—pushing the boundaries of our understanding of cosmic acceleration.

Appendices

Appendix A — Field Equations and Friedmann Modification

This appendix contains the complete mathematical derivations underlying MRET's scalar field dynamics and its coupling to the Friedmann expansion law.

1. **Covariant Action**

$$S = \int \sqrt{-g}[\frac{R}{16_{\pi}G} - \frac{1}{2}(\nabla\phi)^2 - V(\phi) + \phi J_{BH}(x)]d^4x$$

2. **Variation with respect to $\boldsymbol{\phi}$** yields the **field equation**:

$$\Box\phi - V'(\phi) = -J_{BH}(x)$$

3. **Energy–momentum tensor** for the ϕ field:

$$T_{\mu\nu}^{(\phi)} \nabla_{\mu}\phi\nabla_{\nu}\phi - g_{\mu\nu}[\frac{1}{2}(\nabla\phi)^2 + V(\phi)]$$

4. **Modified Friedmann equation** (spatially flat):

$$H^2(z) = H_m^2(z) + A\phi^2(z)$$

Where H_m is the matter-only expansion rate.

Appendix B — BHARD Datasets, Processing, and Uncertainties

- **Sources**: Combined AGN and quasar luminosity functions from multi-survey datasets (e.g., SDSS, COSMOS, GOODS).
- **Conversion to BHARD**:
  - Luminosities converted to accretion rates via bolometric corrections and radiative efficiency assumptions ($\epsilon \approx 0.1$).
  - Comoving volume integration yields the BHARD curve.
- **Smoothing and Error Treatment**:
  - Gaussian process regression to interpolate gaps and suppress noise.
  - Error propagation includes luminosity function uncertainties, bolometric correction variance, and cosmic variance terms.

Appendix C — Simulated $H(z)$and Anisotropy Outputs

- **Synthetic $H(z)$ Generation**:
  - Input BHARD curve into lag–memory–saturation kernel for $\phi(t)$.
  - Compute $H(z)$ via modified Friedmann equation.
  - Add DESI-like statistical noise to mimic real data sampling.
- **Directional Anisotropy Simulation**:
  - Seed ϕ field with anisotropic SMBH overdensity maps.
  - Project onto the sky to produce mock $H0(n^\wedge)$maps.
  - Extract dipole amplitude and compare to current Hubble anisotropy constraints.